DESCRIBING FUNCTIONS   1.1.3 through 1.2.2

In addition to introducing students to the classroom norms of problem-based learning, the main objective of these lessons is for students to be able to fully describe the key elements of the graph of a function. To fully describe the graph of a function, students should respond to these graph investigation questions:

Graph Investigation Question	Sample Summary Statement
What does the graph look like?	*The graph looks like half of a parabola on its side.*
Is the graph increasing or decreasing (reading left to right)?	*As x gets bigger, y gets bigger.*
What are the *x*- and *y*-intercepts?	*The graph intersects both the x- and y-axes at (0, 0).*
Are there any limitations on the inputs (domain) of the equation?	*Only positive values of x and zero are possible.*
Are there any limitations on the outputs (range) of the equation? (Is there a maximum or minimum *y*-value?)	*The smallest y-value is 0. There is no maximum y-value.*
Are there any special points?	*The graph has a "starting" point at (0,0).*
Does the graph have any symmetry? If so, where?	*This graph has no symmetry.*

The more formal concepts of function and domain and range are addressed in Lessons 1.2.4 and 1.2.5.

For more information, see the Math Notes boxes in Lessons 1.1.1, 1.1.2, and 1.1.3. Student responses to the Learning Log in Lesson 1.1.1 (problem 1-32), if it was assigned, can also be helpful.

Example 1

For the function $y = x^2 - 2x - 3$, make an $x \rightarrow y$ table, draw a graph, and fully describe the features.

At this point there is no way to know how many points are sufficient for the $x \rightarrow y$ table. Add more points as necessary until you are convinced of shape and location.

x	–2	–1	0	1	2	3
y	5	0	–3	–4	–3	0

Be careful with substitution and Order of Operations when calculating values. For example if $x = -2$,
$y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$.
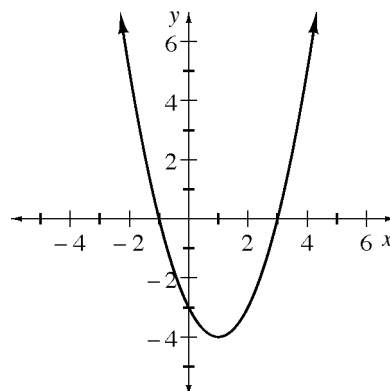
The graph is a parabola; it opens upward. The x-intercepts are $(-1,0)$ and $(3,0)$. The y-intercept is $(0,-3)$. Reading from left to right, the graph decreases until $x = 1$ and then increases. The minimum (lowest) point on the graph (called the vertex) is $(1,-4)$. The vertical line $x = 1$ is a line of symmetry. There are no limitations on inputs to the function. Outputs can be any value greater than or equal to –4.

Example 2

For the function $y = \sqrt{x+3} - 2$, make an $x \rightarrow y$ table, draw a graph, and describe the features.

Note that the smallest possible number for the $x \rightarrow y$ table is $x = -3$. Anything smaller will require the square root of a negative, which is not a real number.

x	–4	–3	0	1	3	6
y		–2	–0.3	0	0.4	1

The graph is half a parabola. It starts at $(-3, -2)$ and has x-intercept $(1, 0)$ and y-intercept $(0, \approx -0.3)$. The graph increases from left to right. The inputs are limited to values of –3 or greater, and the outputs are limited to –2 or greater. There is no line of symmetry.
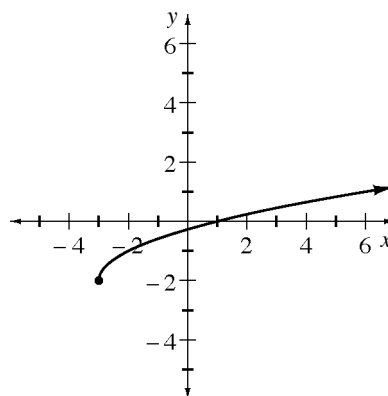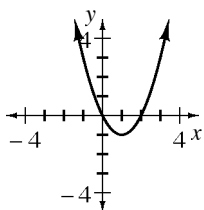
Problems

For each function, make an $x \rightarrow y$ table, draw a graph, and describe the features.

1. $y = x^2 - 2x$

2. $y = x^2 + 2x - 3$

3. $y = \sqrt{x-2}$

4. $y = 4 - x^2$

5. $y = x^2 + 2x + 1$

6. $y = -\sqrt{x} + 3$

7. $y = -x^2 + 2x - 1$
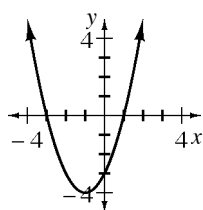
8. $y = |x+2|$

9. $y = 2\sqrt[3]{x} - 1$

2

Answers

1.

Parabola; intercepts $(0, 0), (2, 0)$; decreasing until $x = 1$ then increasing; minimum value at $(1, -1)$; $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to -1.
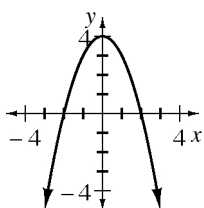
2.

Parabola; intercepts $(-3, 0)$, $(1, 0)$ and $(0, -3)$; decreasing until $x = -1$, then increasing; minimum value at $(-1, -4)$; $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to -4.
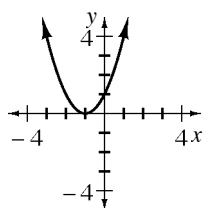
3.
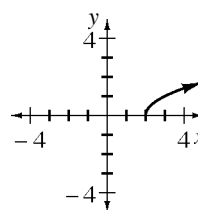
Half-parabola; starting point, intercept and minimum point $(2, 0)$; increasing for $x > 2$. Inputs can be any number greater than or equal to 2. Outputs are greater than or equal to 0.
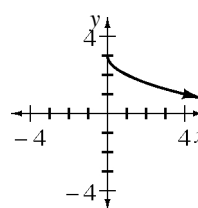
4.

Parabola; intercepts $(-2, 0)$, $(2, 0)$ and $(0, 4)$; increasing for $x < 0$, decreasing for $x > 0$; maximum value at $(0, 4)$; $x = 0$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 4.
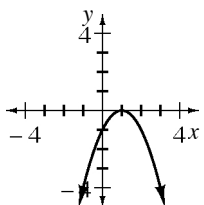
5.

Parabola; intercept $(-1, 0)$; decreasing for $x < -1$, increasing for $x > -1$; minimum value at $(-1, 0)$; $x = -1$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.
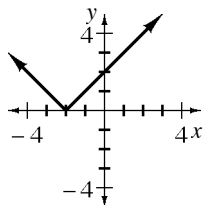
6.

Half-parabola; starting point, intercept and maximum point $(0, 3)$; decreasing for $x > 0$. Inputs can be any number greater than or equal to 0. Outputs are less than or equal to 3.
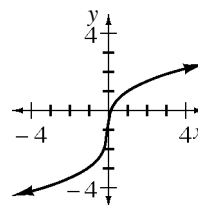
7.

Parabola; intercepts $(1, 0)$ and $(0, -1)$; increasing for $x < 1$, decreasing for $x > 1$; maximum value at $(1, 0)$; $x = 1$ is a line of symmetry. Inputs can be any real number. Outputs are less than or equal to 0.

8.

V-shape; intercepts $(-2, 0)$ and $(0, 2)$; decreasing for $x < -2$, increasing for $x > -2$; minimum value at $(-2, 0)$; $x = -2$ is a line of symmetry. Inputs can be any real number. Outputs are greater than or equal to 0.

9.

S-shape; intercepts $(0, -1)$ and $(0.125, 0)$; increasing for all x from left to right. Inputs and outputs can be any real number. There is no line of symmetry.