FUNCTIONS 1.2.4 through 1.2.5

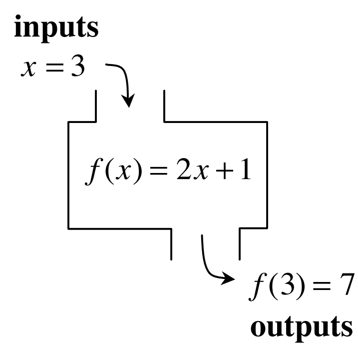
A relationship between the input values (usually *x*) and the output values (usually *y*) is called a function if for each input value, there is no more than one output value. Functions can be represented with an illustration of an input–output "machine," as shown in Lesson 1.2.3 of the textbook and in the diagram in Example 1 below.

The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

For additional information about functions, function notation, and domain and range, see the Math Notes box in Lesson 1.2.5.

Example 1

Numbers, represented by a letter or symbol such as x, are input into the function machine labeled f one at a time, and then the function performs the operation on each input to determine each output, $f(x)$. For example, when $x = 3$ is put into the function f at right, the machine multiplies 3 by 2 and adds 1 to get the output, $f(x)$ which is 7. The notation $f(3) = 7$ shows that the function named f connects the input $x = 3$ with the output 7. This also means the point (3, 7) lies on the graph of the function.



Example 2

a. If $f(x) = \sqrt{x-2}$ then $f(11) = ?$ $\quad f(11) = \sqrt{11-2} = \sqrt{9} = 3$

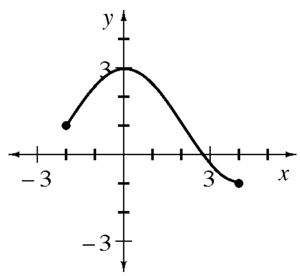
b. If $g(x) = 3 - x^2$ then $g(5) = ?$ $\quad g(5) = 3 - (5)^2 = 3 - 25 = -22$

c. If $f(x) = \dfrac{x+3}{2x-5}$ then $f(2) = ?$ $\quad f(2) = \dfrac{2+3}{2 \cdot 2 - 5} = \dfrac{5}{-1} = -5$

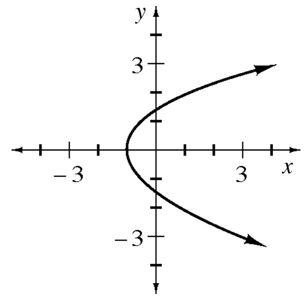
Example 3

A relation in which each input has only one output is called a **function**.

$g(x)$



$f(x)$



$g(x)$ is a function: each input (x) has only one output (y).
$g(-2)=1$, $g(0)=3$, $g(4)=-1$, and so on.

$f(x)$ is not a function: each input greater than -1 has two y-values associated with it.
$f(1)=2$ and $f(1)=-2$.

Example 4

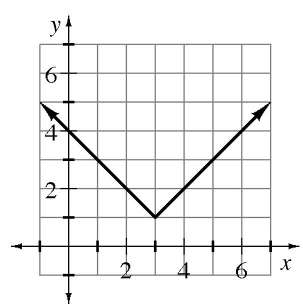
The set of all possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**.

In Example 3 above, the domain of $g(x)$ is $-2 \le x \le 4$, or "all numbers between -2 and 4." The range is $-1 \le y \le 3$ or "all numbers between -1 and 3."

The domain of $f(x)$ in Example 3 above is $x \ge -1$ or "any real number greater than or equal to -1," since the graph starts at -1 and continues forever to the right. Since the graph of $f(x)$ extends in both the positive and negative y directions forever, the range is "all real numbers."

Example 5

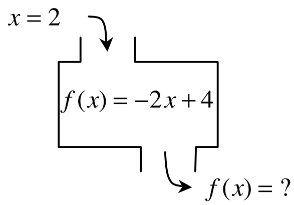
For the graph at right, since the x-values extend forever in both directions the domain is "all real numbers." The y-values start at 1 and go higher so the range is $y \ge 1$ or "all numbers greater or equal to 1."



Problems

Determine the outputs for the given inputs of the following functions.

1.



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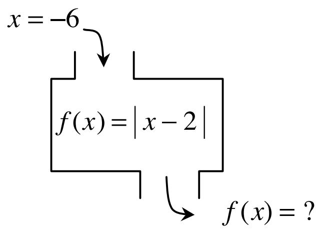

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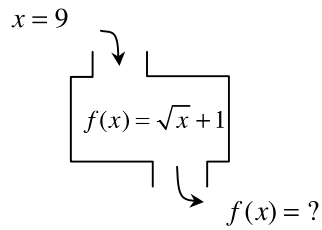


4. $f(x) = (5 - x)^2$
$f(8) = ?$

5. $g(x) = x^2 - 5$
$g(-3) = ?$

6. $f(x) = \frac{2x+7}{x^2-9}$
$f(3) = ?$

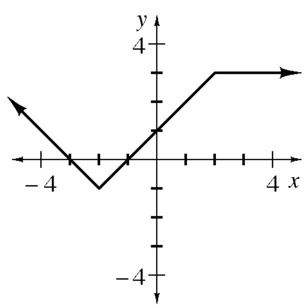
7. $h(x) = 5 - \sqrt{x}$
$h(9) = ?$

8. $h(x) = \sqrt{5 - x}$
$h(9) = ?$

9. $f(x) = -x^2$
$f(4) = ?$

Determine if each relation is a function. Then state its domain and range.

10.



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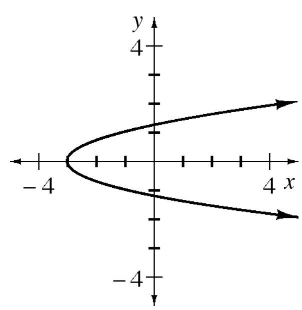

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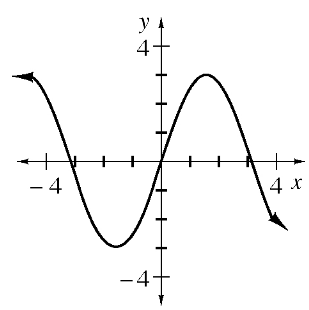

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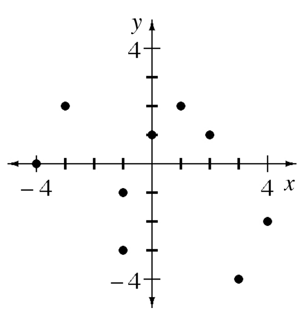

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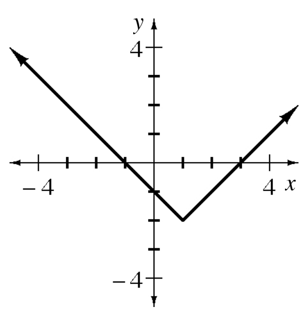

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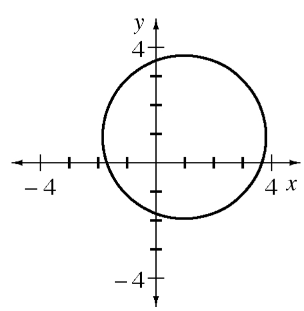

Answers

1. $f(2) = 0$
2. $f(-6) = 8$
3. $f(9) = 4$
4. $f(8) = 9$
5. $g(-3) = 4$
6. not possible
7. $f(9) = 2$
8. not possible
9. $f(4) = -16$
10. Yes, each input has one output; domain is all numbers, range is $-1 \le y \le 3$
11. No, for example $x = 0$ has two outputs; domain is $x \ge -3$, range is all numbers
12. Yes; domain all numbers, range is $-3 \le y \le 3$
13. No; $x = -1$ has two outputs; domain is $-4, -3, -1, 0, 1, 2, 3, 4$, range is $-4, -3, -2, -1, 0, 1, 2$
14. Yes; domain is all numbers, range is $y \ge -2$
15. No, many inputs have two outputs; domain is $-2 \le x \le 4$ range is $-2 \le y \le 4$