CURVED BEST-FIT MODELS 6.2.5

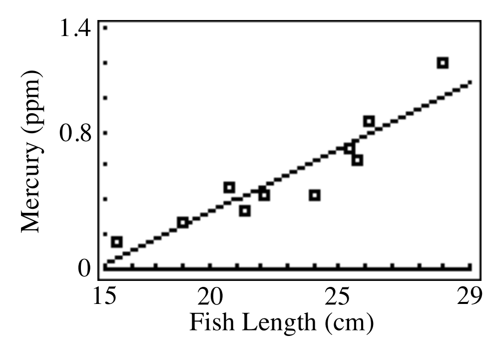
Many relationships are modeled better by a curve than by a line. For additional information, see the Math Notes box in Lesson 6.2.5.

Example

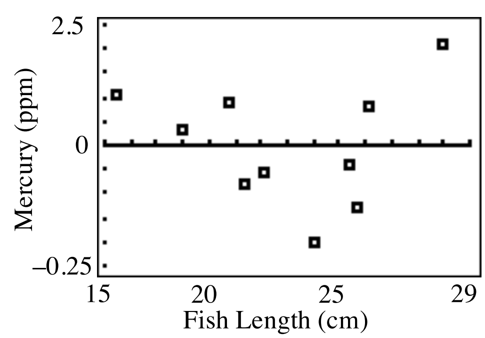
Methyl mercury is a neurotoxin found in many fish species. Plankton absorb mercury from contaminated runoff water, then small fish eat the plankton and the methyl mercury travels up the food web to larger predators. In general, the larger the fish, the higher the concentration of methyl mercury in its flesh. Assume a biologist is sampling white bass from a lake known to have mercury in the water in an effort to find a relationship between the length of the fish and the concentration of methyl mercury in the fish. The following data was collected from a sample of 10 white bass (ppm is "parts per million"):

Length, cm	24.4	23.0	21.1	24.7	25.1	20.3	18.0	27.9	19.8	15.4
Mercury, ppm	0.689	0.428	0.425	0.629	0.863	0.340	0.281	1.20	0.474	0.164

The LSRL was calculated and the following scatterplot and residual plots were made:



The curved pattern in the residual plot indicates that a non-linear model would be a better fit.



Example continues on next page →

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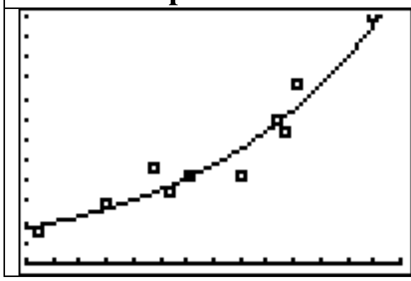
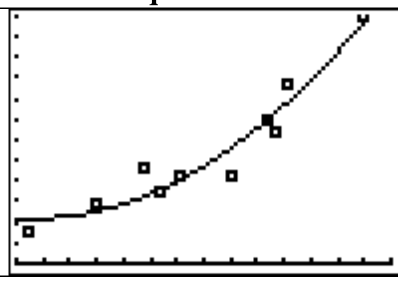
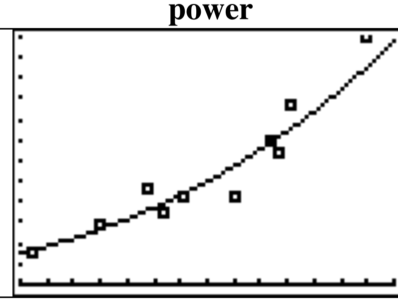
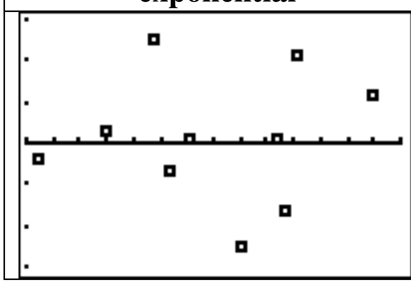
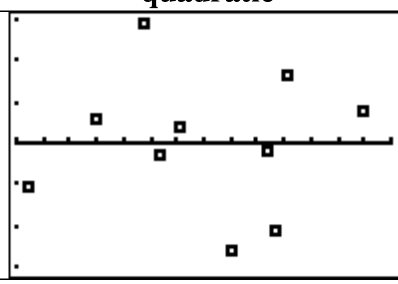
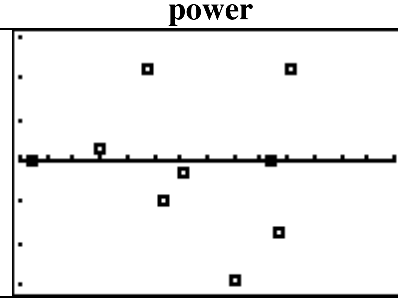
Example continued from previous page.

a. Perform exponential, quadratic, and power model regressions for the data and make the associated scatter and residual plots. These regressions are all available on the TI-83/84+ calculator on the same screen as the linear regression.

SCATTERPLOTS

exponential	**quadratic**	**power**
		

RESIDUAL PLOTS

exponential	**quadratic**	**power**
		

b. Use the residual plots to select the best non-linear model.

Try to find the model with the least patterning or the most “random”-looking scatter. The power model still has a U-shaped pattern; the power model is not a good choice.

Both the exponential and quadratic look fairly random; either would be an appropriate choice based on the plots. But successive growth like this—in this case, the buildup of mercury as the fish ages and thus gets longer—is more likely explained by an exponential relationship than a quadratic one. The exponential model makes a good choice to describe the association between length of the fish and the concentration of mercury.

c. Use your model to predict the amount of mercury in a 20-cm fish.

$y = 0.0187 \cdot 1.1588^x$ where x is the length of the fish in cm, and y is the concentration of mercury in parts per million. If $x = 20$ cm, then $y = 0.356$. A 20-cm fish is expected to contain 0.356 ppm of mercury.

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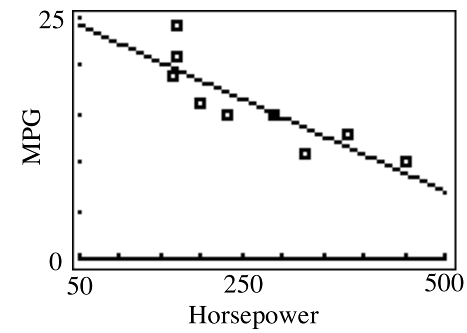
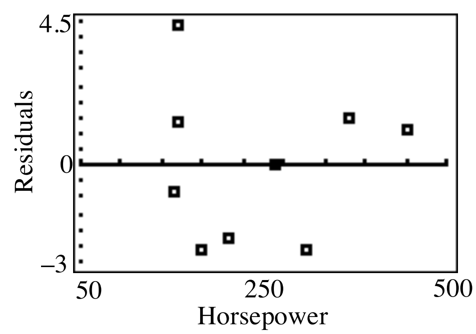
Problem

It seems reasonable that the horsepower of a car is related to its gas mileage. Suppose a random sample of 9 car models is selected and the engine horsepower and city gas mileage is recorded for each one.

HP	197	170	166	230	381	170	326	451	290
MPG	16	24	19	15	13	21	11	10	15

The LSRL was calculated and the following scatter and residual plots were made. (These plots were also created in Lesson 6.2.1 of this *Parent Guide with Extra Practice*.)





The curved pattern in the residual plot indicates that a non-linear model would be a better fit.

a. Perform exponential, quadratic, and power model regressions for the data and make the associated scatter and residual plots. These regressions are all available on the TI-83/84+ calculator on the same screen as the linear regression.

b. Use the residual plots to select the best non-linear model.

c. What do you predict the gas mileage will be for a car with 400 horsepower?

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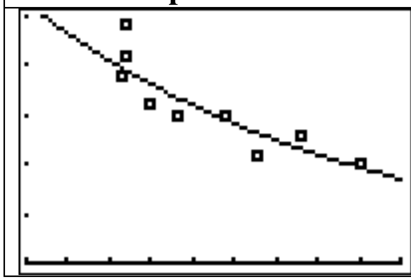
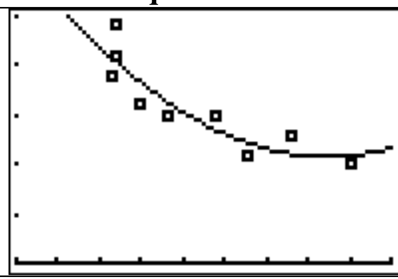
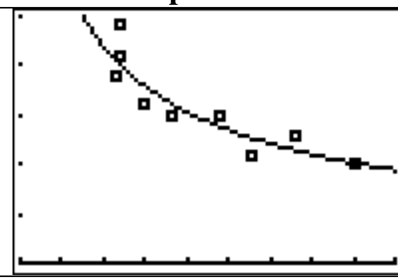
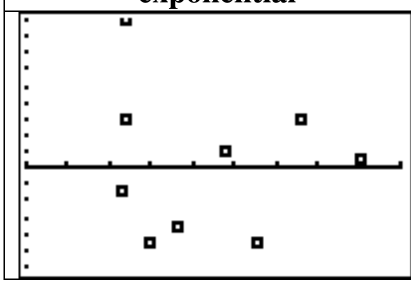
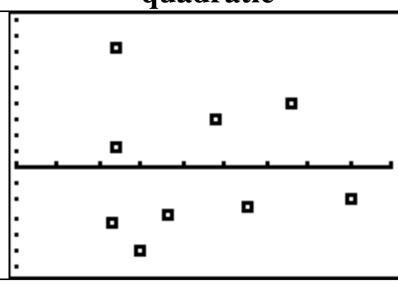
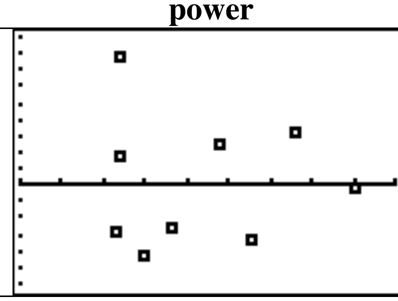
Answer

a.

SCATTERPLOTS

exponential	quadratic	power
		

RESIDUAL PLOTS

exponential	quadratic	power
		

b. The exponential residual plot still seems a little U-shaped. Both the quadratic and power models appear to have more randomly scattered residual plots. However we have some very serious concerns about the quadratic model: the quadratic model turns back up, indicating that for slightly higher horsepower the gas mileage increases. That makes no sense. We will use the power model to represent this data.

c. For the power model, $y = 732.94 \cdot x^{-0.7004}$, where x is the horsepower and y is the gas mileage (in mpg). If $x = 400$, then $y = 11.03$. We predict that a car with 400 horsepower will get about 11 miles per gallon.

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