ELIMINATION METHOD 4.2.3 through 4.2.5

In previous work with systems of equations, one of the variables was usually alone on one side of one of the equations. In those situations, it is convenient to rewrite both equations in $y = mx + b$ form and use the Equal Values Method, or to substitute the expression for one variable into the other equation using the Substitution Method.

Another method for solving systems of equations is the **Elimination Method**. This method is particularly convenient when both equations are in standard form (that is, $ax + by = c$). To solve this type of system, we can rewrite the equations by focusing on the coefficients. (The coefficient is the number in front of the variable.)

See problem 4-56 in the textbook for an additional explanation of the Elimination Method.

For additional information, see the Math Notes boxes in Lessons 4.2.4 and 5.1.1. For additional examples and more problems solving systems using multiple methods, see the Checkpoint 7B materials in the back of the textbook.

Example 1

Solve: $x - y = 2$

$2x + y = 1$

Recall that you are permitted to add the same expression to both sides of an equation. Since $x - y$ is equivalent to 2 (from the first equation), you are permitted to add $x - y$ to one side of the second equation, and 2 to the other side. Then solve.

$$\begin{aligned} 2x + y &= 1 \\ +x - y &\quad +2 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

Note that this was an effective way to eliminate y and find x because $-y$ and y were opposite terms; $y + (-y) = 0$.

Now substitute the value of x in either of the original equations to find the y-value.

$$\begin{aligned} 2x + y &= 1 \\ \text{Since } x &= 1, \\ 2(1) + y &= 1 \\ 2 + y &= 1 \\ y &= -1 \end{aligned}$$

The solution is $(1, -1)$, since $x = 1$ and $y = -1$ make both of the original equations true. On a graph, the point of intersection of the two original lines is $(1, -1)$. Always check your solution by substituting the solution back into *both* of the original equations to verify that you get true statements.

See problem 4-56 in the textbook for an additional explanation of the Elimination Method.

Example 2

Solve: $3x + 6y = 24$

$3x + y = -1$

Notice that both equations contain a $3x$ term. We can rewrite $3x + y = -1$ by multiplying both sides by -1, resulting in $-3x + (-y) = 1$. Now the two equations have terms that are opposites: $3x$ and $-3x$. This will be useful in the next step because $-3x + 3x = 0$.

Since $-3x + (-y)$ is equivalent to 1, we can add $-3x + (-y)$ to one side of the equation and add 1 to the other side.

$$\begin{array}{rcl} 3x + 6y & = & 24 \\ -3x + (-y) & & +1 \\ 5y & = & 25 \\ & y = 5 & \end{array}$$

Notice how the two opposite terms, $3x$ and $-3x$, eliminated each other, allowing us to solve for y.

Then substitute the value of y into either of the original equations to find x.

$$\begin{aligned} 3x + 6(5) &= 24 \\ 3x + 30 &= 24 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$

The solution is $(-2, 5)$. It makes both equations true, and names the point where the two lines intersect on a graph. Always check your solution by substituting the solution back into *both* of the original equations to verify that you get true statements.

A more detailed explanation of this method can be found in the following example.

Example 3

To use the Elimination Method, one of the terms in one of the equations needs to be opposite of the corresponding term in the other equation. One of the equations can be multiplied to make terms opposite. For example, in the system at right, there are no terms that are opposite. However, if the first equation is multiplied by -4, then the two equations will have $4x$ and $-4x$ as opposites. The first equation now looks like this: $-4(x + 3y = 7) \rightarrow -4x + (-12y) = -28$. When multiplying an equation, be sure to multiply all the terms on *both* sides of the equation. With the first equation rewritten, the system of equations now looks like this:

$$\begin{aligned} x + 3y &= 7 \\ 4x - 7y &= -10 \end{aligned}$$

$$\begin{aligned} -4x + (-12y) &= -28 \\ 4x - 7y &= -10 \end{aligned}$$

Since $4x - 7y$ is equivalent to -10, they can be added to both sides of the first equation:

$$\begin{array}{rrcr} & -4x + (-12y) & = & -28 \\ + & 4x - 7y & & -10 \\ \hline & -19y & = & -38 \\ & y & = & 2 \end{array}$$

Now any of the equations can be used to find x:

Since $4x - 7y = -10$ and $y = 2$,

$$\begin{aligned} 4x - 7(2) &= -10 \\ 4x - 14 &= -10 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

The solution to the system of equations is $(1, 2)$.

Example 4

If multiplying one equation by a number will not make it possible to eliminate a variable, multiply both equations by different numbers to get coefficients that are the same or opposites.

Solve: $8x - 7y = 5$
$3x - 5y = 9$

One possibility is to multiply the first equation by 3 and the second equation by –8. The resulting terms 24x and –24x will be opposites, setting up the Elimination Method.

$$3(8x - 7y = 5) \Rightarrow 24x - 21y = 15$$
$$-8(3x - 5y = 9) \Rightarrow -24x + 40y = -72$$

The system of equations is now:

$$\begin{aligned} 24x - 21y &= 15 \\ -24x + 40y &= -72 \end{aligned}$$

This system can be solved by adding equivalent expressions (from the second equation) to the first equation:

$$\begin{array}{rrl} & 24x - 21y & = 15 \\ + & -24x + 40y & \;\; -72 \\ \hline & 19y & = -57 \\ & y & = -3 \end{array}$$

Then solving for x, the solution is (–2, –3).

For an additional example like this one (where both equations had to be multiplied to create opposite terms), see Example 2 in the Checkpoint 7B materials at the back of the textbook.

Example 5

The special cases of "no solution" and "infinite solutions" can also occur. See the Math Notes box in Lesson 4.2.5 for additional information.

Solve: $4x + 2y = 6$
$2x + y = 3$

Multiplying the second equation by 2 produces $4x + 2y = 6$. The two equations are identical, so when graphed there would be one line with *infinite* solutions because the same ordered pairs are true for both equations.

Solve: $2x + y = 3$
$4x + 2y = 8$

Multiplying the first equation by 2 produces $4x + 2y = 6$. There are no numbers that could make $4x + 2y$ equal to 6, and $4x + 2y$ equal to 8 at the same time. The lines are parallel and there is *no solution*, that is, no point of intersection.

SUMMARY OF METHODS TO SOLVE SYSTEMS

Method	This Method is Most Efficient When	Example
Equal Values	Both equations in y-form.	$y = x - 2$ $y = -2x + 1$
Substitution	One variable is alone on one side of one equation.	$y = -3x - 1$ $3x + 6y = 24$
Elimination: Add to eliminate one variable.	Equations in standard form with opposite coefficients.	$x + 2y = 21$ $3x - 2y = 7$
Elimination: Multiply one equation to eliminate one variable.	Equations in standard form. One equation can be multiplied to create opposite terms.	$x + 2y = 3$ $3x + 2y = 7$
Elimination: Multiply both equations to eliminate one variable.	When nothing else works. In this case you could multiply the first equation by 3 and the second equation by –2, then add to eliminate the opposite terms.	$2x - 5y = 3$ $3x + 2y = 7$

Problems

1. $2x + y = 6$, $-2x + y = 2$
2. $-4x + 5y = 0$, $-6x + 5y = -10$
3. $2x - 3y = -9$, $x + y = -2$
4. $y - x = 4$, $2y + x = 8$
5. $2x - y = 4$, $\frac{1}{2}x + y = 1$
6. $-4x + 6y = -20$, $2x - 3y = 10$
7. $6x - 2y = -16$, $4x + y = 1$
8. $6x - y = 4$, $6x + 3y = -16$
9. $2x - 2y = 5$, $2x - 3y = 3$
10. $y - 2x = 6$, $y - 2x = -4$
11. $4x - 4y = 14$, $2x - 4y = 8$
12. $3x + 2y = 12$, $5x - 3y = -37$

Answers

1. $(1, 4)$
2. $(5, 4)$
3. $(-3, 1)$
4. $(0, 4)$
5. $(2, 0)$
6. Infinite
7. $(-1, 5)$
8. $(-\frac{1}{6}, -5)$
9. $(4.5, 2)$
10. No solution
11. $(3, -\frac{1}{2})$
12. $(-2, 9)$