

In these sections, students generalize what they have learned about geometric sequences to investigate exponential functions. Students study exponential functions of the form $y = ab^x$. Students look at multiple representations of exponential functions, including graphs, tables, equations, and context. They learn how to move from one representation to another. Students learn that the value of a is the “starting value” of the function— a is the y -intercept or the value of the function at $x = 0$. b is the growth (multiplier). If $b > 1$ then the function increases; if b is a fraction between 0 and 1 (that is, $0 < b < 1$), then the function decreases (decays). In this course, values of $b < 0$ are not considered.

For additional information, see the Math Notes boxes in Lesson 7.1.3 and 7.2.3. For additional examples and more practice, see the Checkpoint 9 and Checkpoint 10A materials at the back of the student textbook.

Example 1

LuAnn has \$500 with which to open a savings account. She can open an account at Fredrico’s Bank, which pays 7% interest, compounded monthly, or Money First Bank, which pays 7.25%, compounded quarterly. LuAnn plans to leave the money in the account, untouched, for ten years. In which account should she place the money? Justify your answer.

Solution: The obvious answer is that she should put the money in the account that will pay her the most interest over the ten years, but which bank is that? At both banks the principal (the initial value) is \$500. Fredrico’s Bank pays 7% compounded monthly, which means the interest rate is $\frac{0.07}{12} \approx 0.00583$ each month. If LuAnn puts her money into Fredrico’s Bank, after one month she will have:

$$500 + 500(0.00583) = 500(1.00583) \approx \$502.92.$$

To calculate the amount at the end of the second month, we must multiply by 1.00583 again, making the amount:

$$500(1.00583)^2 \approx \$505.85.$$

At the end of three months, the balance is:

$$500(1.00583)^3 \approx \$508.80.$$

This will happen every month for ten years, which is 120 months. At the end of 120 months, the balance will be:

$$500(1.00583)^{120} \approx \$1004.43.$$

Note that this last equation is an exponential function in the form $y = ab^x$, where y is the amount of money in the account and x is the number of months (in this case, 120 months). $a = 500$ is the starting value (at 0 months), and $b = 1.00583$ is the multiplier or growth rate for the account each month.

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A similar calculation is performed for Money First Bank. Its interest rate is higher, 7.25%, but it is only calculated and compounded quarterly. (Quarterly means four times each year, or every three months.) Hence, every quarter the bank calculates $\frac{0.0725}{4} = 0.018125$ interest. At the end of the first quarter, LuAnn would have:

$$500(1.018125) \approx \$509.06.$$

At the end of ten years (40 quarters) LuAnn would have:

$$500(1.018125)^{40} \approx \$1025.69.$$

Note that this last equation is an exponential function in the form $y = ab^x$, where y is the amount of money in the account and x is the number of quarters (in this case, 40 quarters). $a = 500$ is the starting value (at 0 quarters), and $b = 1.018125$ is the multiplier or growth rate for the account each quarter.

Since Money First Bank would pay her approximately \$21 more in interest than Fredrico's Bank, she should put her money in Money First Bank.

Example 2

Most homes appreciate in value, at varying rates, depending on the home's location, size, and other factors. But if a home is used as a rental, the Internal Revenue Service allows the owner to assume that it will depreciate in value. Suppose a house that costs \$150,000 is used as a rental property, and depreciates at a rate of 8% per year. What is the multiplier that will give the value of the house after one year? What is the value of the house after one year? What is the value after ten years? When will the house be worth half of its purchase price? Draw a graph of this situation.

Solution: Unlike interest, which increases the value of the house, depreciation takes value away. After one year, the value of the house is $150000 - 0.08(150000)$ which is the same as $150000(0.92)$. Therefore the multiplier is 0.92. After one year, the value of the house is $150000(0.92) = \$138,000$. After ten years, the value of the house will be $150000(0.92)^{10} \approx \$65,158.27$.

This last equation is an exponential function in the form $y = ab^x$, where y is the value of the house and x is the number of years. $a = 150000$ is the starting value (at 0 years), and $b = 0.92$ is the multiplier or growth factor (in this case, decay) each year.

To find when the house will be worth half of its purchase price, we need to determine when the value of the house reaches \$75,000. We just found that after ten years, the value is below \$75,000, so this situation occurs in less than ten years. To help answer this question, list the house's values in a table to see the depreciation.

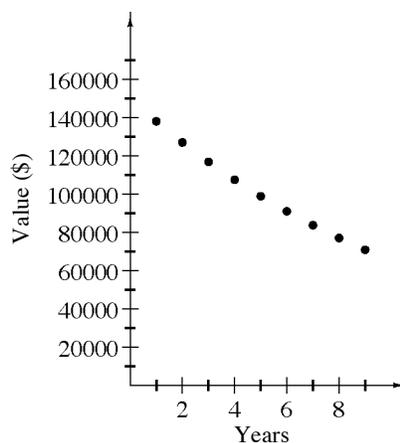
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From the table or graph, you can see that the house will be worth half its purchase price after 8 years.

Note: You can write the equation $75000 = 150000 \cdot 0.92^x$, but you will not have the mathematics to solve this equation for x until a future course. However, you can use the equation to find a more exact value: try different values for x in the equation, so the y -value gets closer and closer to \$75,000. At about 8.313 years the house's value is close to \$75,000.

# Years	House's value
1	138000
2	126960
3	116803.20
4	107458.94
5	98862.23
6	90953.25
7	83676.99
8	76982.83
9	70824.20



Example 3

Write an equation that represents the function in this table.

Week	Weight of Bacterial Culture (g)
1	756.00
2	793.80
3	833.49

The exponential function will have the form $y = ab^x$, where y is the weight of the bacterial culture, and x is the number of weeks. The multiplier, b , for the weight of the bacterial culture is 1.05 (because $793.80 \div 756 = 1.05$ and $833.49 \div 793.80 = 1.05$, etc.). The starting point, a is not given because we are not given the weight at Week 0. However, since the growth is 1.05 every week, we know that $(1.05) \cdot (\text{weight at Week 0}) = 756.00\text{g}$. The weight at Week 0 is 720g, thus $a = 720$. We can now write the equation:

$$y = 720 \cdot 1.05^x,$$

where y is the weight of the bacterial culture (g), and x is the time (weeks).

Problems

- In seven years, Seta's son Stu is leaving home for college. Seta hopes to save \$8000 to pay for his first year. She has \$5000 now and has found a bank that pays 7.75% interest, compounded daily. At this rate, will she have the money she needs for Stu's first year of college? If not, how much more does she need?
- Eight years ago, Rudi thought that he was making a sound investment by buying \$1000 worth of Pro Sports Management stock. Unfortunately, his investment depreciated steadily, losing 15% of its value each year. How much is the stock worth now? Justify your answer.
- Based on each table below, find the equation of the exponential function $y = ab^x$.

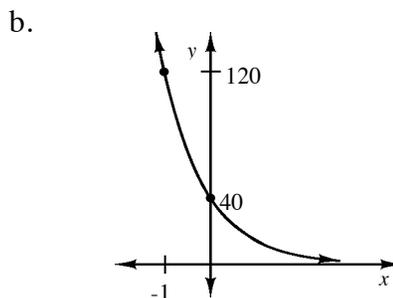
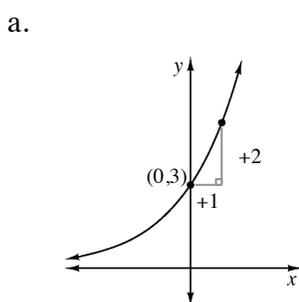
a.

x	$f(x)$
0	1600
1	2000
2	2500
3	3125

b.

x	y
1	40
2	32
3	25.6

- The new Bamo Super Ball has a rebound ratio of 0.97. If you dropped the ball from a height of 125 feet, how high will it bounce on the tenth bounce?
- Based on each graph below, find the equation of the exponential function $y = ab^x$.



- Fredrico's Bank will let you decide how often your interest will be computed, but with certain restrictions. If your interest is compounded yearly you can earn 8%. If your interest is compounded quarterly, you earn 7.875%. Monthly compounding earns a 7.75% interest rate, while weekly compounding earns a 7.625% interest rate. If your interest is compounded daily, you earn 7.5%. What is the best deal? Justify your answer.
- Fully investigate the graph of the function $y = \left(\frac{3}{4}\right)^x + 4$. See Describing Functions (Lessons 1.1.3 through 1.2.2) in this *Parent Guide with Extra Practice* for information on how to fully describe the graph of a function.

Answers

1. Yes, she will have about \$8601.02 by then. The daily rate is $\frac{0.0775}{365} \approx 0.000212329$. Seven years is 2555 days, so we have $\$5000(1.000212329)^{2555} \approx \8601.02 .
2. It is now only worth about \$272.49.
3. a. $y = 1600(1.25)^x$ b. $y = 50(0.8)^x$
4. About 92.18 feet.
5. a. $y = 3\left(\frac{5}{3}\right)^x$ b. $y = 40\left(\frac{1}{3}\right)^x$
6. The best way to do this problem is to choose any amount, and see how it grows over the course of one year. Taking \$100, after one year, 8% compounded yearly will yield \$108. 7.875% compounded quarterly yields \$108.11. 7.75% compounded monthly yields \$108.03. 7.625% compounded weekly yields \$107.91. 7.5% compounded daily yields \$107.79. Quarterly is the best.
7. This is a function that it is continuous and nonlinear (curved). It has a y-intercept of (0, 5), and no x-intercepts. The domain is all real values of x , and the range is all real values of $y > 4$. This function has a horizontal asymptote of $y = 4$, and no vertical asymptotes. It is an exponential function.