

FACTORIZING QUADRATICS

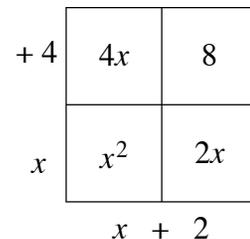
8.1.1 through 8.1.4

Chapter 8 introduces students to rewriting quadratic expressions and solving quadratic equations. Quadratic functions are functions which can be rewritten in the form $y = ax^2 + bx + c$ (where $a \neq 0$) and when graphed, create a U-shaped curve called a parabola.

There are multiple methods that can be used to solve quadratic equations. One of them requires factoring the quadratic expression first. In Lessons 8.1.1 through 8.1.4, students factor quadratic expressions.

In previous chapters, students used algebra tiles to build “generic rectangles” of quadratic expressions. In the figure below, the length and width of the rectangle are $(x + 2)$ and $(x + 4)$. Since the area of a rectangle is given by $(\text{base})(\text{height}) = \text{area}$, the area of the rectangle in the figure below can be expressed as a *product*, $(x + 2)(x + 4)$. But the small pieces of the rectangle also make up its area, so the area can be expressed as a *sum*, $4x + 8 + x^2 + 2x$, or $x^2 + 6x + 8$. Thus students wrote $(x + 2)(x + 4) = x^2 + 6x + 8$.

In the figure at right, the length and width of the rectangle, which are $(x + 2)$ and $(x + 4)$, are *factors* of the quadratic expression $x^2 + 6x + 8$, since $(x + 2)$ and $(x + 4)$ multiply together to produce the quadratic expression $x^2 + 6x + 8$. Notice that the $4x$ and the $2x$ are located diagonally from each other. They are like terms and can be combined and written as $6x$.



The factors of $x^2 + 6x + 8$ are $(x + 2)$ and $(x + 4)$.

The ax^2 term and the c term are always diagonal to one another in a generic rectangle. In this example, the ax^2 term is $(1x^2)$ and the c term is the constant 8; the product of this diagonal is $1x^2 \cdot 8 = 8x^2$. The two x -terms make up the other diagonal and can be combined into a sum since they are like terms. The b of a quadratic expression is the *sum* of the coefficients of these factors: $2x + 4x = 6x$, so $b = 6$. The product of this other diagonal is $(2x)(4x) = 8x^2$. *Note that the products of the two diagonals are always equivalent.* In the textbook, students may nickname this rule “Casey’s Rule,” after the fictional character Casey in problem 8-4.

To factor a quadratic expression, students need to identify the coefficients of the two x -terms so that the products of the two diagonals are equivalent, and also the sum of the two x -terms is b . Students can use a “diamond problem” to help organize their sums and products. For more information on using a diamond problem and generic rectangle to factor quadratic expressions, see the Math Notes box in Lesson 8.1.4.

For additional information, see the Math Notes boxes in Lessons 8.1.1 through 8.1.4. For additional examples and more practice, see the Checkpoint 10B materials at the back of the student textbook.

Example 1

Factor $x^2 + 7x + 12$.

Sketch a generic rectangle.

Place the x^2 and the 12 along one diagonal.

	12
x^2	

Find two terms whose product is $12x^2$ and whose sum is $7x$.

In this case, $3x$ and $4x$. (Students are familiar with this situation as a “diamond problem” from Chapter 1.)

3x	12
x^2	4x

Write these terms along the other diagonal. Either term can go in either diagonal space.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and finding the greatest common factor of each row and column.

+ 3	3x	12
x	x^2	4x
	x + 4	

Write the sum as a product (factored form).

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Example 2

Factor $x^2 + 7x - 30$.

Sketch a generic rectangle.

Place the x^2 and the -30 along one diagonal.

	-30
x^2	

Find two terms whose product is $-30x^2$ and whose sum is $7x$. In this case, $-3x$ and $10x$.

-3x	-30
x^2	10x

Write these terms along the other diagonal. Either term can go in either diagonal space.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and finding the greatest common factor of each row and column.

-3	-3x	-30
x	x^2	10x
	x + 10	

Write the sum as a product (factored form).

$$x^2 + 7x - 30 = (x - 3)(x + 10)$$

Example 3

Factor $x^2 - 15x + 56$.

Sketch a generic rectangle.

Place the x^2 and the 56 along one diagonal.

Find two terms whose product is $56x^2$ and whose sum is $-15x$. Write these terms as the other diagonal.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and finding the greatest common factor of each row and column.

Write the sum as a product (factored form).

	56
x^2	

$-8x$	56
x^2	$-7x$

-8	$-8x$	56
x	x^2	$-7x$
	$x - 7$	

$$x^2 - 15x + 56 = (x - 7)(x - 8)$$

Example 4

Factor $12x^2 - 19x + 5$.

Sketch a generic rectangle.

Place the $12x^2$ and the 5 along one diagonal.

Find two terms whose product is $60x^2$ and whose sum is $-19x$. Write these terms as the other diagonal.

Find the base and height of the rectangle. Check the signs of the factors.

Write the sum as a product (factored form). $(3x - 1)(4x - 5) = 12x^2 - 19x + 5$

$-15x$	5		
$12x^2$	$-4x$	-5	5
		$4x$	
		$3x - 1$	

Example 5

Factor $3x^2 + 21x + 36$.

Note: If a common factor appears in all the terms, it should be factored out first.

For example, $3x^2 + 21x + 36 = 3(x^2 + 7x + 12)$.

Then $x^2 + 7x + 12$ can be factored in the usual way, as in Example.

$$x^2 + 7x + 12 = (x + 3)(x + 4).$$

Then, since the expression $3x^2 + 21x + 36$ has a factor of 3,

$$3x^2 + 21x + 36 = 3(x^2 + 7x + 12) = 3(x + 3)(x + 4).$$

Problems

- | | | | |
|-------------------------|------------------------|----------------------|----------------------|
| 1. $x^2 + 5x + 6$ | 2. $2x^2 + 5x + 3$ | 3. $3x^2 + 4x + 1$ | 4. $3x^2 + 30x + 75$ |
| 5. $x^2 + 15x + 44$ | 6. $x^2 + 7x + 6$ | 7. $2x^2 + 22x + 48$ | 8. $x^2 + 4x - 32$ |
| 9. $4x^2 + 12x + 9$ | 10. $24x^2 + 22x - 10$ | 11. $x^2 + x - 72$ | 12. $3x^2 - 20x - 7$ |
| 13. $x^3 - 11x^2 + 28x$ | 14. $2x^2 + 11x - 6$ | 15. $2x^2 + 5x - 3$ | 16. $x^2 - 3x - 10$ |
| 17. $4x^2 - 12x + 9$ | 18. $3x^2 + 2x - 5$ | 19. $6x^2 - x - 2$ | 20. $9x^2 - 18x + 8$ |

Answers

- | | | | |
|------------------------|-------------------------|------------------------|------------------------|
| 1. $(x + 2)(x + 3)$ | 2. $(x + 1)(2x + 3)$ | 3. $(3x + 1)(x + 1)$ | 4. $3(x + 5)(x + 5)$ |
| 5. $(x + 11)(x + 4)$ | 6. $(x + 6)(x + 1)$ | 7. $2(x + 8)(x + 3)$ | 8. $(x + 8)(x - 4)$ |
| 9. $(2x + 3)(2x + 3)$ | 10. $2(3x - 1)(4x + 5)$ | 11. $(x - 8)(x + 9)$ | 12. $(x - 7)(3x + 1)$ |
| 13. $x(x - 4)(x - 7)$ | 14. $(x + 6)(2x - 1)$ | 15. $(x + 3)(2x - 1)$ | 16. $(x - 5)(x + 2)$ |
| 17. $(2x - 3)(2x - 3)$ | 18. $(3x + 5)(x - 1)$ | 19. $(2x + 1)(3x - 2)$ | 20. $(3x - 4)(3x - 2)$ |