

FRACTIONAL EXPONENTS**7.2.1**

A fractional exponent is equivalent to an expression with roots or radicals.

$$\text{For } x \neq 0, x^{a/b} = (x^a)^{1/b} = \sqrt[b]{x^a} \text{ or } x^{a/b} = (x^{1/b})^a = (\sqrt[b]{x})^a.$$

Fractional exponents may also be used to solve equations containing exponents. For additional information, see the Math Notes box in Lesson 7.2.2.

Example 1

Rewrite each expression in radical form and simplify if possible.

a. $16^{5/4}$

b. $(-8)^{2/3}$

Solution:

a. $16^{5/4}$	OR	a. $16^{5/4}$
$= (16^{1/4})^5$		$= (16^5)^{1/4}$
$= (\sqrt[4]{16})^5$		$= (1,048,576)^{1/4}$
$= (2)^5$		$= \sqrt[4]{1,048,576}$
$= 32$		$= 32$

b. $(-8)^{2/3}$	OR	b. $(-8)^{2/3}$
$= ((-8)^{1/3})^2$		$= ((-8)^2)^{1/3}$
$= (\sqrt[3]{-8})^2$		$= (64)^{1/3}$
$= (-2)^2$		$= \sqrt[3]{64}$
$= 4$		$= 4$

Example 2

Simplify each expression. Each answer should contain no parentheses and no negative exponents.

a. $(144x^{-12})^{1/2}$

b. $\left(\frac{8x^7y^3}{x}\right)^{-1/3}$

Using the Power Property of Exponents, the Property of Negative Exponents, and the Property of Fractional Exponents:

a. $(144x^{-12})^{1/2} = \left(\frac{144}{x^{12}}\right)^{1/2}$
 $= \sqrt{\frac{144}{x^{12}}} = \frac{12}{x^6}$

b. $\left(\frac{8x^7y^3}{x}\right)^{-1/3} = \left(\frac{x}{8x^7y^3}\right)^{1/3}$
 $= \left(\frac{1}{8x^6y^3}\right)^{1/3} = \sqrt[3]{\frac{1}{8x^6y^3}} = \frac{1}{2x^2y}$

Example 3

Solve the following equations for x .

a. $x^7 = 42$

b. $3x^{12} = 132$

Solution: As with many equations, we need to isolate the variable (get the variable by itself), and then eliminate the exponent. This will require one of the Laws of Exponents, namely $(x^a)^b = x^{ab}$.

a. $x^7 = 42$

$$(x^7)^{1/7} = (42)^{1/7}$$

$$x^{7 \cdot (1/7)} = (42)^{1/7}$$

$$x^1 = (42)^{1/7}$$

$$x \approx 1.706$$

b. $3x^{12} = 132$

$$\frac{3x^{12}}{3} = \frac{132}{3}$$

$$x^{12} = 44$$

$$(x^{12})^{1/12} = (44)^{1/12}$$

$$x = (44)^{1/12}$$

$$x \approx \pm 1.371$$

The final calculation takes the seventh root of 42 in part (a) and the twelfth root of 44 in part (b). Notice that there is only one answer for part (a), where the exponent is odd, but there are two answers (\pm) in part (b) where the exponent is even. Even roots always produce two answers, a positive and a negative. Be sure that if the problem is a real-world application that both the positive and the negative results make sense before stating both as solutions. You may have to disregard one solution so that the answer is feasible.

Problems

Change each expression to radical form and simplify.

1. $(64)^{2/3}$

2. $16^{-1/2}$

3. $(-27)^{1/3}$

Simplify the following expressions as much as possible.

4. $(16a^8b^{12})^{3/4}$

5. $\frac{144^{1/2}x^{-3}}{(16^{3/4}x^7)^0}$

6. $\frac{a^{2/3}b^{-3/4}c^{7/8}}{a^{-1/3}b^{1/4}c^{1/8}}$

Solve the following equations for x .

7. $x^8 = 65,536$

8. $-5x^{-3} = \frac{25}{40}$

9. $3^{5x} = 9^{x-1}$

10. $\left(\frac{2}{3}\right)^x = 1$

11. $2(3x-5)^4 = 512$

12. $2^{4x-1} = 4$

Find the error in each of the following solutions. Then give the correct solution.

13. $4(x+7)^6 = 1392$

14. $5^{4x+2} = 10^{3x-1}$

$$(x+7)^6 = 348$$

$$5^{4x+2} = 2 \cdot 5^{3x-1}$$

$$x+7 = 58$$

$$4x+2 = 2(3x-1)$$

$$x = 51$$

$$4x+2 = 6x-1$$

$$3 = 2x$$

$$x = 1.5$$

Answers

1. $(\sqrt[3]{64})^2 = 16$

2. $\frac{1}{\sqrt{16}} = \frac{1}{4}$

3. $\sqrt[3]{-27} = -3$

4. $8a^6b^9$

5. $\frac{12}{x^3}$

6. $\frac{ac^{3/4}}{b}$

7. $x = 4$

8. $x = -2$

9. $x = -\frac{2}{3}$

10. $x = 0$

11. $x = 3$

12. $x = \frac{3}{4}$

13. Both sides need to be raised to the $\frac{1}{6}$ (or the 6th root taken), not divided by six. $x \approx -4.35$.

14. Since 5 and 10 cannot be written as the power of the same number, the only way to solve the equation now is by guess and check. $x \approx 11.75$. If you did not get this answer, do not worry about it now. The point of the problem is to spot the error. There is a second error: the 2 was not distributed in the fourth line.