INTRODUCTION TO SEQUENCES 5.1.1 through 5.1.3

In Chapter 5, students investigate sequences by looking for patterns and rules. Initially in the chapter, students concentrate on arithmetic sequences (sequences generated by adding a constant to the previous term), and then later in the chapter (and in Chapter 7) they consider geometric sequences (sequences generated by multiplying the previous term by a constant).

In Lessons 5.1.1 through 5.1.3 students are introduced to the two types of sequences, arithmetic and geometric, and their graphs, in everyday situations.

For additional examples and explanations, see the next section of this *Parent Guide with Extra Practice*, "Equations for Sequences." For additional information, see the first half of the Math Notes box in Lesson 5.3.2.

Example 1

Peachy Orchard Developers are preparing land to create a large subdivision of single-family homes. They have already built 15 houses on the site. Peachy Orchard plans to build six new homes every month. Create a table of values that will show the number of houses in the Peachy Orchard subdivision over time. Write an equation relating the number of houses over time. Graph the sequence.

Since the subdivision initially has 15 homes, 15 is the number of homes at time $t = 0$. After one month, there will be six more, or 21 homes. After the second month, there will be 27 homes. After each month, we add six homes to the total number of homes in the subdivision. Because we are adding a constant amount after each time period, this is an **arithmetic sequence**.

n, the number of months	$t(n)$, the total number of homes
0	15
1	21
2	27
3	33
4	39

We can find the equation for this situation by noticing that this is a linear function: the growth is constant. All arithmetic sequences are linear.

One way to write the equation that models this situation is to notice that the slope (growth) = 6 homes/month, and the y-intercept (starting point) = 15. Then in $y = mx + b$ form, the equation is $y = 6x + 15$.

Another way to find the equation of a line, especially in situations more complex than this one, is to use two points on the line, calculate the slope (m) between the two points, and then solve for the y-intercept (as in Lesson 2.3.2). This method is shown in the following steps:

Example continues on next page →

Example continued from previous page.

Choose $(1, 21)$ and $(4, 39)$

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$

$m = \frac{39-21}{4-1}$

$m = \frac{18}{3}$

$m = 6$

$y = mx + b$

at $(x, y) = (1, 21)$ and $m = 6$,

$21 = 6(1) + b$

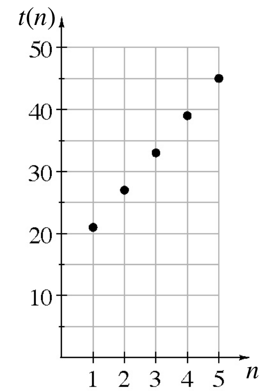
$b = 15$

$y = 6x + 15$

We write the equation as $t(n) = 6n + 15$ to show that this is an arithmetic sequence (as opposed to the linear function $y = mx + b$ or $f(x) = mx + b$) that will find the term t, for any number n. Let $t(n)$ represent the number of houses, and n the number of months.

The sequence would be written: $21, 27, 33, 39, \ldots$. Note that sequences usually begin with the first term (in this case, the term for the first month, $n = 1$).

The graph for the sequence is shown at right. Note that it is linear, and that it starts with the point $(1, 21)$.



Example 2

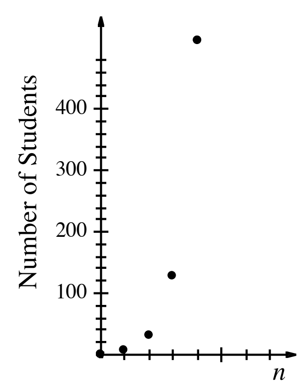
When Rosa tripped and fell into a muddy puddle at lunch (she was so embarrassed!), she knew exactly what would happen: within ten minutes, the two girls who saw her fall would each tell four people what they had seen. Within the next ten minutes, those eight students would each tell four more people. Rosa knew this would continue until everyone in the entire school was talking about her muddy experience. If there are 2016 students in the school, how many "generations" of gossiping would it take before everyone in the school was talking about Rosa? How many minutes would it take? Graph the situation.

At time $t = 0$, only two people see Rosa trip and fall. After ten minutes, each of those two people would tell four people; there are eight students gossiping about Rosa. After another ten minutes, each of those eight students will gossip with four more students; there will be $8 \times 4 = 32$ students gossiping. For the third increment of ten minutes, each of the 32 students will gossip with 4 students; $32 \times 4 = 128$ students gossiping.

Each time we multiply the previous number of students by four to get the next number of students. This is an example of a **geometric sequence**, and the multiplier is four. We record this in a table as shown at right, with n being the number of ten minute increments since Rosa's fall, and $t(n)$ is the number of students discussing the incident at that time. By continuing the table, we note that at time $t = 6$, there will be 2048 students discussing the mishap. Since there are only 2016 students in the school, everyone is gossiping by the sixth ten-minute increment of time. Therefore, just short of 60 minutes, or a little before one hour, everyone knows about Rosa's fall in the muddy puddle.

n, Number of Ten Minute Increments	Number of Students
0	2
1	8
2	32
3	128
4	512
5	1024
6	2048

The graph is shown at right. A geometric relationship is not linear; it is exponential. In future lessons, students would write the sequence as 8, 32, 128, … . Note that sequences usually begin with the first term (in this case, the term for the first month, $n = 1$).



Problems

1. Find the missing terms for this arithmetic sequence and an equation for $t(n)$.

$$\_\_, 15, 11, \_\_, 3$$

2. For this sequence each term is $\frac{1}{5}$ of the previous one. Work forward and backward to find the missing terms.

$$\_\_, \_\_, \frac{2}{3}, \_\_, \_\_$$

3. The 30th term of a sequence is 42. If each term in the sequence is four greater than the previous number, what is the first term?

4. The microscopic length of a crystalline structure grows so that each day it is 1.005 times as long as the previous day. If on the third day the structure was 12.5 nm long, write a sequence for how long it was on the first five days. (nm stands for nanometer, or 1×10^{-9} meters.)

5. Davis loves to ride the mini-cars at the amusement park but riders must be no more than 125 cm tall. If on his fourth birthday he is 94 cm tall and grows approximately 5.5 cm per year, at what age will he no longer be able to go on the mini-car ride?

Answers

1. 19 and 8; $t(n)=23-4n$

2. $\frac{50}{3}, \frac{10}{3}, \frac{2}{3}, \frac{2}{15}, \frac{2}{75}$

3. $42-29(4)=-74$

4. $\approx12.38, \approx12.44, 12.5, \approx12.56, \approx12.63, \ldots$

5. $t(n)=5.5n+94$, so solve $5.5n+94\le125$. $n\approx5.64$. At $\approx4+5.64=9.64$ he will be too tall. Davis can continue to go on the ride until he is about $9\frac{1}{2}$ years old.