

WRITING AN EQUATION GIVEN THE SLOPE AND A POINT ON THE LINE

2.3.1

In earlier work students used substitution in equations like $y = 2x + 3$ to find x and y pairs that make the equation true. Students recorded those pairs in a table, and then used them as coordinates to graph a line. Every point (x, y) on the line makes the equation true.

Later, students used the patterns they saw in the tables and graphs to recognize and write equations in the form of $y = mx + b$. The “ b ” represents the y -intercept of the line, the “ m ” represents the slope, while x and y represent the coordinates of any point on the line. Each line has a unique value for m and a unique value for b , but there are infinite (x, y) values for each linear equation.

The slope of the line is the same between any two points on that line. We can use this information to write equations without creating tables or graphs.

For additional information, see the Math Notes boxes in Lessons 2.2.2 and 2.2.3.

Example 1

What is the equation of the line with a slope of 2 that passes through the point $(10, 17)$?

Write the general equation of a line.

$$y = mx + b$$

Substitute the values we know: m , x , and y .

$$17 = 2(10) + b$$

Solve for b .

$$17 = 20 + b$$

$$-3 = b$$

Write the complete equation using the values $m = 2$ and $b = -3$.

$$y = 2x - 3$$

Example 2

This algebraic method can help us write equations of parallel lines. Parallel lines never intersect or meet. They have the *same* slope, m , but *different* y -intercepts, b .

What is the equation of the line parallel to $y = 3x - 4$ that goes through the point $(2, 8)$?

Write the general equation of a line.

$$y = mx + b$$

Substitute the values we know: m , x , and y .

Since the lines are parallel, the slopes are equal.

$$8 = 3(2) + b$$

$$8 = 6 + b$$

Solve for b .

$$2 = b$$

Write the complete equation.

$$y = 3x + 2$$

Problems

Write the equation of the line with the given slope that passes through the given point.

1. slope = 5, $(3, 13)$

2. slope = $-\frac{5}{3}$, $(3, -1)$

3. slope = -4 , $(-2, 9)$

4. slope = $\frac{3}{2}$, $(6, 8)$

5. slope = 3, $(-7, -23)$

6. slope = 2, $(\frac{5}{2}, -2)$

Write the equation of the line *parallel* to the given line that goes through the given point.

7. $y = \frac{3}{5}x + 2$ $(0, 0)$

8. $y = 4x - 1$ $(-2, -6)$

9. $y = -2x + 5$ $(-4, -2)$

10. $y = 4x + 5$ $(-6, -28)$

11. $y = \frac{1}{3}x - 1$ $(6, 9)$

12. $y = 3x + 8$ $(0, \frac{1}{2})$

Answers

1. $y = 5x - 2$

2. $y = -\frac{5}{3}x + 4$

3. $y = -4x + 1$

4. $y = \frac{3}{2}x - 1$

5. $y = 3x - 2$

6. $y = 2x - 7$

7. $y = \frac{3}{5}x$

8. $y = 4x + 2$

9. $y = -2x - 10$

10. $y = 4x - 4$

11. $y = \frac{1}{3}x + 7$

12. $y = 3x + \frac{1}{2}$