WRITING EQUATIONS 4.1.1

In this lesson, students translate written information, often modeling everyday situations, into algebraic symbols and linear equations. Students use "let" statements to specifically define the meaning of each of the variables they use in their equations.

For additional examples and more problems, see the Checkpoint 7A problems at the back of the textbook.

Example 1

The perimeter of a rectangle is 60 cm. The length is 4 times the width. Write one or more equations that model the relationships between the length and width.

Start by identifying what is unknown in the situation. Then define variables, using "let" statements, to represent the unknowns. When writing "let" statements, the units of measurement must also be identified. This is often done using parentheses, as shown in the "let" statements below. In this problem, length and width are unknown.

Let w represent the width (cm) of the rectangle, and let l represent the length (cm).

In this problem there are two variables. To be able to find unique solutions for these two variables, two unique equations need to be written.

From the first sentence and our knowledge about rectangles, the equation $P = 2l + 2w$ can be used to write the equation $60 = 2l + 2w$. From the sentence "the length is 4 times the width" we can write $l = 4w$.

A system of equations is two or more equations that use the same set of variables to represent a situation. The system of equations that represent the situation is:

Let w represent the width (cm) of the rectangle, and let l represent the length (cm).

$$l = 4w$$

$$2l + 2w = 60$$

Note that students who took a CPM middle school course may recall a method called the 5-D Process. This 5-D Process is not reviewed in this course, but it is perfectly acceptable for students to use it to help write and solve equations for word problems.

Using a 5-D table:

	Define		**Do**	**Decide**
	Width	Length	Perimeter	$P = 60$?
Trial 1:	10	4(10)	2(40) + 2(10) = 100	too big
Trial 2:	5	4(5)	2(20) + 2(5) = 50	too small
	l	$4(l)$	$2(4w) + 2w = 60$	

Example 2

Mike spent \$11.19 on a bag containing red and blue candies. The bag weighed 11 pounds. The red candy costs \$1.29 a pound and the blue candy costs \$0.79 a pound. How much red candy did Mike have?

Start by identifying the unknowns. The question in the problem is a good place to look because it often asks for something that is unknown. In this problem, the amount of red candy and the amount of blue candy are unknown.

Let r represent the amount of red candy (lb), and b represent the amount of blue candy (lb).

Note how the units of measurement were defined. If we stated "r = red candy" it would be very easy to get confused as to whether r represented the *weight* of the candy or the *cost* of the candy.

From the statement "the bag weighed 11 pounds" we can write $r + b = 11$. Note that in this equation the units are lb + lb = lb, which makes sense.

The cost of the red candy will be \$1.29/pound multiplied by its weight, or $1.29r$. Similarly, the cost of the blue candy will be $0.79b$. Thus $1.29r + 0.79b = 11.19$.

Let r represent the weight of the red candy (lb), and let b represent the weight of the blue candy (lb).

$$r + b = 11$$

$$1.29r + 0.79b = 11.19$$

Problems

Write an equation or a system of equations that models each situation. Do not solve your equations.

1. A rectangle is three times as long as it is wide. Its perimeter is 36 units. Find the length of each side.

2. A rectangle is twice as long as it is wide. Its area is 72 square units. Find the length of each side.

3. The sum of two consecutive odd integers is 76. What are the numbers?

4. Nancy started the year with $425 in the bank and is saving $25 a week. Seamus started the year with $875 and is spending $15 a week. When will they have the same amount of money in the bank?

5. Oliver earns $50 a day and $7.50 for each package he processes at Company A. His paycheck on his first day was $140. How many packages did he process?

6. Dustin has a collection of quarters and pennies. The total value is $4.65. There are 33 coins. How many quarters and pennies does he have?

7. A one-pound mixture of raisins and peanuts costs $7.50. The raisins cost $3.25 a pound and the peanuts cost $5.75 a pound. How much of each ingredient is in the mixture?

8. An adult ticket at an amusement park costs $24.95 and a child's ticket costs $15.95. A group of 10 people paid $186.50 to enter the park. How many were adults?

9. Katy weighs 105 pounds and is gaining 2 pounds a month. James weighs 175 pounds and is losing 3 pounds a month. When will they weigh the same amount?

10. Harper Middle School has 125 fewer students than Holmes Junior High. When the two schools are merged there are 809 students. How many students attend each school?

Answers (Other equivalent forms are possible.)

	One Variable Equation	System of Equations	Let Statement
1.	$2w + 2(3w) = 36$	$l = 3w$ $2w + 2l = 36$	Let $l =$ length, $w =$ width
2.	$w(2w) = 72$	$l = 2w$ $lw = 72$	Let $l =$ length, $w =$ width
3.	$m + (m + 2) = 76$	$m + n = 76$ $n = m + 2$	Let m = the first odd integer and n = the next consecutive odd integer
4.	$425 + 25x = 875 - 15x$	$y = 425 + 25x$ $y = 875 - 15x$	Let x = the number of weeks and y = the total money in the bank
5.	$50 + 7.5p = 140$		Let p = the number of packages Oliver processed
6.	$0.25q + 0.01(33 - q) = 4.65$	$q + p = 33$ $0.25q + 0.01p = 4.65$	Let q = number of quarters, p = number of pennies
7.	$3.25r + 5.75(1 - r) = 7.5$	$r + p = 1$ $3.25r + 5.75p = 7.5(1)$	Let r = weight of raisins and p = weight of peanuts
8.	$24.95a + 15.95(10 - a) = 186.5$	$a + c = 10$ $24.95a + 15.95c = 186.5$	Let a = number of adult tickets and c = number of child's tickets
9.	$105 + 2m = 175 - 3m$	$w = 105 + 2m$ $w = 175 - 3m$	Let m = the number of months and w = the weight of each person
10.	$x + (x - 125) = 809$	$x + y = 809$ $y = x - 125$	Let x = number of Holmes students and y = number of Harper students